Simple unconventional geometric scenario of one-way quantum computation with superconducting qubits inside a cavity

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We propose a simple unconventional geometric scenario to achieve a kind of nontrivial multi-qubit operations with superconducting charge qubits placed in a microwave cavity. The proposed quantum operations are insensitive not only to the thermal state of cavity mode but also to certain random operation errors, and thus may lead to high-fidelity quantum information processing. Executing the designated quantum operations, a class of highly entangled cluster states may be generated efficiently in the present scalable solid-state system, enabling one to achieve one-way quantum computation.

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Quantum computers have been paid much attention for the past decade, as they may accomplish certain tough tasks that can hardly be fulfilled by their classical counterpart. Despite rather advanced theoretical concepts of quantum computation, practical physical implementation appears to be at an early stage. Recently, superconducting qubits have attracted significant interests because of their potential suitability for scalable quantum computation [1]. The realization of an entangled state of two qubits [2, 3] and implementation of a conditional phase gate operation [4] were reported in this scalable solid state system. Note that, superconducting qubits are quite sensitive to the external environment and the background noise, with the decoherence time being rather short. In order to couple multipartite qubits, a lot of auxiliary devices are often needed, which increases the complexity of circuits and also inevitably introduces additional uncontrollable noises, and thus would make the fidelity and scalability of the system no longer better. On the other hand, the cavity quantum electrodynamics (QED), which addresses the properties of atoms coupled to discrete photon modes in high Q cavities, was also proposed as a potential setup for quantum information processing including quantum computation [5]. In sharp contrast to the above resource-consuming coupling, the cavity mode can act as a ”bus” and thus easily mediate a kind of long range interactions among the superconducting qubits [6, 7, 8, 9, 10]. In this sense, an idea to place superconducting qubits in the cavity (i. e., the superconducting cavity QED) is more promising for quantum computation, being a macroscopic analogy of atomic quantum computing and control. This quantum setup not only provides strong inhibition of spontaneous emission, which leads to the enhancement of qubit lifetimes, but also suppresses greatly the decoherence caused by the external environment since the cavity may serve as a magnetic shield.

In this paper, we propose a simpler scheme for implementing a kind of unconventional geometric phase gates [11] with superconducting charge qubits coupled to a microwave cavity mode [10]. The proposed quantum operations depend only on global geometric features [12] and are insensitive to the thermal state of cavity modes. Thus they may lead to the high-fidelity quantum information processing. More importantly, executing the designated quantum operations, a class of highly entangled cluster states may be generated efficiently and thus a new kind of one-way quantum computation scheme can be achieved.

A single superconducting qubit considered here, as shown in Fig (1a), consists of a small superconducting box with excess Cooper-pair charges, formed by an symmetric superconducting quantum interference device (SQUID) with the capacitance C_J and Josephson coupling energy E_J, pierced by an external magnetic flux Φ. A control gate voltage V_g is connected to the system via a gate capacitor C_g. The Hamiltonian of the system is [1]

$$H = E_c(n-\bar{n})^2 - E_J\cos\varphi_1 - E_J\cos\varphi_2, \qquad (1)$$

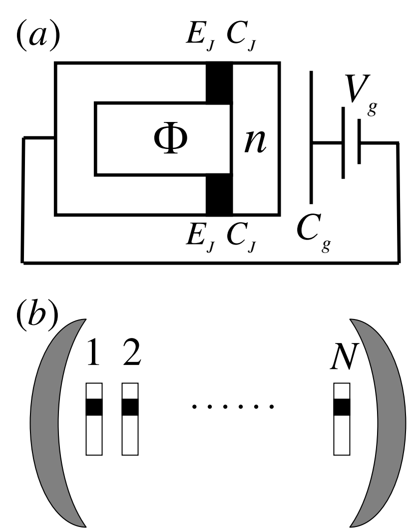

FIG. 1: (a) A schematic superconducting charge qubit connected to a SQUID loop consisting of two identical Josephson junctions, subject to a controllable gate voltage V_g and a magnetic flux Φ. (b) N qubits are placed in a microwave cavity, where the qubit-qubut couplings are mediated by the cavity mode.

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where n is the number operator of (excess) Cooper-pair charges on the box, $E_c = 2e^2/(C_g + 2C_J)$ is the charging energy, $\bar{n} = C_g V_g/2$ is the induced charge controlled by the gate voltage V_g, and φ_m ($m = 1, 2$) is the gauge-invariant phase difference between the two sides of the mth junction. We here focus on the charging regime where $E_c \gg E_J$. In this case, a convenient basis is formed by the charge states, parameterized by the number of Cooper pairs n on the box with its conjugate φ; they satisfy the standard commutation relation: $[\varphi, n] = i$. At temperatures much lower than the charging energy and restricting the gate charge to the range of $N_g \in [0, 1]$, only a pair of adjacent charge states $\{|0\rangle, |1\rangle\}$ on the island are relevant. The Hamiltonian (1) is then reduced to

$$H = -E_{ce}\sigma_z - E(\Phi)\sigma_x, \tag{2}$$

where $E_{ce} = 2E_c(1 - 2\bar{n})$, $E(\Phi) = E_J \cos(\pi \frac{\Phi}{\phi_0})$, σ_x and σ_z are the Pauli matrices. Clearly, two noncommuting single-qubit gates σ_x and σ_z can be obtained directly from this Hamiltonian by simply tuning the gate voltage close to the degeneracy point ($\bar{n} \sim 1/2$) or adjusting the external flux Φ. For a system of N independent qubits with each being at the degeneracy point and Φ being time-independent, the time evolution of such system may be written as

$$U(t)_1 = \exp\left[i\frac{E(\Phi)}{\hbar}t\sum_{j=1}^{N}\sigma_x^j\right]. \tag{3}$$

On the other hand, let us consider a superconducting qubit to be placed in a cavity as shown in Fig. (1b). The gauge-invariant phase difference is

$$\varphi_m^{'} = \varphi_m - \frac{2\pi}{\phi_0}\int_{l_m}\mathbf{A}_m \cdot d\mathbf{l}_m,$$

where $\mathbf{A}_m$ is the vector potential in the same gauge of φ_m. $\mathbf{A}_m$ may be divided into two parts $\mathbf{A}_m' + \mathbf{A}_m^{\phi}$, where the first and second terms arise respectively from the electromagnetic field of the cavity normal modes and the external magnetic flux, respectively. For simplicity, we here assume that the cavity has only a single mode to play a role. In the Coulomb gauge, $\mathbf{A}_m'$ takes the form $\sqrt{\hbar/2\omega_c V}(a + a^\dagger)\hat{\epsilon}$ [6], where $\hat{\epsilon}$ is the unit polarization vector of cavity mode, V is the volume of the cavity, a and $a^\dagger$ are the annihilation and creation operators for the quantum oscillators, and ω_c is its frequency. Therefore, we have

$$\frac{2\pi}{\phi_0}\int_{l_m}\mathbf{A}_m \cdot d\mathbf{l}_m = \frac{2\pi}{\phi_0}\int_{l_m}\mathbf{A}_m^{\phi} \cdot d\mathbf{l}_m + g(a + a^\dagger),$$

where $\phi_0 = \pi\hbar/e$ being the flux quantum, $g = 2e\hat{\epsilon} \cdot \mathbf{l}/\sqrt{2\varepsilon\omega_c V\hbar}$ is the coupling constant between the junctions and the cavity, with l the thickness of the insulating layer in the junction. The closed path integral of the $\mathbf{A}^{\phi}$ gives rise to the magnetic flux: $\oint_C \mathbf{A}^{\phi} d\mathbf{l} = \Phi$. Similar to that in Ref. [10], setting $\pi\Phi/\phi_0 = \omega t$ with $\omega_c - \omega = \delta \ll \omega$, the Hamiltonian (1) reads

$$H = E_c(n - \bar{n})^2\sigma_z - \frac{E_J}{2}\left(\sigma^\dagger e^{-i[\frac{g}{2}(a+a^\dagger)+\omega t]} + \text{H.c.}\right). \tag{4}$$

Consider that N such qubits are located within a single-mode cavity. To a good approximation, the whole system can be considered as N two-level systems coupled to a quantum harmonic oscillator [6]. Setting the qubits at their degeneracy points, the system considered here can then be described by the Hamiltonian $H = H_0 + H_{int}$, where

$$H_0 = \hbar\omega_c\left(a^\dagger a + \frac{1}{2}\right), \tag{5a}$$

$$H_{int} = -\frac{E_J}{2}\sum_{j=1}^{N}\left(\sigma_j^+ e^{-i[\frac{g}{2}(a+a^\dagger)+\omega t]} + \text{H.c.}\right). \tag{5b}$$

The spin notation is used for the qubit j with Pauli matrices $\{\sigma_j^x, \sigma_j^y, \sigma_j^z\}$, and $\sigma_j^{\pm} = (\sigma_j^x \pm i\sigma_j^y)/2$. For simplicity, we have also assumed the same E_c and E_J for all qubits. Expanding the Hamiltonian (5b) to the first order of g in the Lamb-Dicke limit and under the rotating wave approximation as well as in the interaction picture $U_0 = \exp(-iH_0 t)$, the Hamiltonian is reduced to

$$H_{int} = \frac{igE_J}{4}\left(a^\dagger e^{i\delta t} - ae^{-i\delta t}\right)J_x, \tag{6}$$

where $J_x = \sum_{j=1}^{n}\sigma_j^x$. The time-evolution operator for Hamiltonian (6) can be expressed as

$$\begin{aligned} U(t) &= \exp\left\{\left[\int_0^t B_{(t)}^* dB_{(t)}\right]J_x^2\right\} \\ &\quad \times \exp\left[iB_{(t)}^* aJ_x\right]\exp\left[iB_{(t)}a^\dagger J_x\right], \end{aligned} \tag{7}$$

where $B_{(t)} = (gE_J/4\hbar\delta)(1 - e^{i\delta t})$. Setting $\delta T = 2k\pi$ ($k = 1, 2, \cdots$) leads to

$$U(\gamma) = \exp(i\gamma J_x^2) \tag{8}$$

with

$$\gamma = \left(\frac{gE_J}{4\hbar\delta}\right)^2\delta T. \tag{9}$$

Interestingly, this $U(\gamma)$-operation is insensitive to the thermal state of the cavity mode as the related influence represented by the last two exponents in Eq. (7) is completely removed.

It is also notable that, when only two qubits are considered in Eq. (8), it is straightforward to check that $U(\gamma)$ is a non-trivial two-qubit unconventional geometric phase gate [11], where the phase γ satisfies the relation $\gamma = \gamma_g + \gamma_d = -\gamma_g$ (i.e., $\gamma_d = -2\gamma_g$), with γ_g and γ_d being respectively the geometric and dynamic phases accumulated in the evolution; this unconventional geometric phase shift still depends only on global geometric features and is robust against random operation errors [11], thus the high-fidelity of the two-qubit operation may be experimentally achieved. For example, as an entangling operation gate, it can entangle two qubits from a separable state to a fully entangled EPR-state

$$|00\rangle \xrightarrow{U(\gamma)} \frac{1}{\sqrt{2}}(|00\rangle + i|11\rangle), \tag{10}$$

once we set $\gamma = \pi/8$. More generally, the operation (8) can be used to generate multipartite entangled GHZ state with an extended unconventional geometric phase shift scenario in this system [10]. In addition, this solid-state architecture may also provide an alternative geometric approach to construct quantum error correcting code [10].

In most current efforts, the universal quantum computation is achieved with sequences of controlled interactions between selected qubits. Being significantly different from these efforts, Raussendorf and Briegel [13] proposed a new kind of scalable quantum computation, namely the one-way quantum computation, which constructs quantum logic gates by single-qubit measurements on cluster states. The distinct advantage of one-way computing strategy lies in that it separates the processes of generating entanglement and executing computation. So one can tolerate failures during the generation process simply by repeating the process, provided that the failures are heralded. Due to its novel application in quantum computing, the generation of cluster states has also been proposed in the context of the cavity QED with atomic qubits [14] and superconducting qubits [15]. However, all these generation schemes are of the "step by step" nature, which means that the time needed for generating the cluster states is determined by the number of the qubits, thus the scalability of these schemes is unlikely good. Very recently, Tanamoto *et al*. [16] proposed a new scheme to generate the cluster state of superconducting qubits by only one step. However, since the inter-qubit interaction was capacitively coupled in the scheme [16], where each qubit works far away from the degeneracy point, the decoherence time of a single qubit is much shorter than that at the degeneracy point. In addition, the capacitive inter-qubit coupling is fixed [3], thus it is difficult to prepare the initial state for each qubit. You *et al*. [17] proposed another scenario to improve the performance of generating cluster states by introducing an inductive inter-qubit coupling. We note that both schemes need significant resources to couple different qubits [16, 17].

As a direct and useful application of the multi-qubit operator (8), we here present an efficient way for generating the multipartite cluster states. In the present scalable solid-state system, the effective long range couplings among qubits are mediated by the cavity field, and thus no auxiliary devices are needed. The operator (8) is equivalent to

$$U(\gamma)_2 = \exp\left(i2\gamma \sum_{j>i=1}^{n} \sigma_x^i \sigma_x^j\right), \tag{11}$$

up to an overall phase factor. Since the two operators (3) and (11) commute with each other, we can perform the two corresponding operations with the time intervals t_1 and T sequentially to obtain the wanted unitary operation [18]. Setting

$$2E(\Phi)t_1 = (N-1)\hbar\gamma, \tag{12}$$

we have the total evolution operator as

$$U(t_1+T) = \exp\left[i8\gamma\left(\sum_{j>i=1}^{N} \frac{1+\sigma_x^i}{2}\frac{1+\sigma_x^j}{2}\right)\right]. \tag{13}$$

The initial state of each charge qubit can be prepared as

$$|0\rangle_i = \frac{1}{\sqrt{2}}\left(|-\rangle_i + |+\rangle_i\right),$$

where $|\pm\rangle_i = (|0\rangle_i \mp |1\rangle_i)/\sqrt{2}$ are eigenstates of $H_i = -E(\Phi)\sigma_x^{(i)}$ with eigenvalues $\pm E(\Phi)$. When the condition $\gamma = (2n+1)\pi/8$ is satisfied, the generated cluster state is

$$\frac{1}{2^{N/2}}\bigotimes_{i=1}^{N}\left(|-\rangle_i(-1)^{N-i}\prod_{j=i+1}^{N}\sigma_x^{(j)} + |+\rangle_i\right), \tag{14}$$

which is a highly entangled state. The operator $\sigma_x^{(j)}$ acts on the states $|\pm\rangle$ of the qubits $j = i+1, \ldots, N$, with $i = 1, 2, \ldots, N-1$; this is due to the inter-qubit coupling mediated by cavity field is of the long-range nature. The above condition can be satisfied whenever

$$\delta = \frac{gE_J}{\hbar}\sqrt{\frac{k}{2n+1}}. \tag{15}$$

Correspondingly, the requirement (12) may be expressed as

$$t_1 = \frac{(N-1)(2n+1)\pi\hbar}{16E(\Phi)}. \tag{16}$$

In the present proposal for producing cluster states, an effective anisotropic direction is along the x-axis, rather than the z-direction for the standard Ising model [13]. Now the cluster state is represented using the eigenstates of σ_x, and correspondingly, the single-qubit projective measurements are performed on the eigenstates of σ_z.

Comparing with the existing schemes for generating cluster states, our scheme possesses likely the following advantages.

(1) As a macroscopic analogy of atomic quantum computing and control, the present one provides strong inhibition of spontaneous emission and suppresses the decoherence caused by the external environment. The positions of qubits in the cavity are fixed and thus they can be easily and selectively addressed. It seems easy to scale up to large number of qubits with the present one, and the control and measurement techniques are more advanced for this system.

(2) The present scheme works at the degeneracy point, where the qubit has a longer decoherence time. The initialization and operation (3) for the qubits can be easily achieved. After generating the cluster state, no external magnetic flux is applied and the inter-qubit coupling is also switched off. This is convenient for implementing one-way quantum computation via local single-qubit measurements, which can be more efficiently implemented, e.g., using a single-electron transistor coupled to the charge qubit [1].

(3) Our scheme is quite simple and feasible in comparison with that addressed in Ref. [10], where the superconducting qubit was formed by two symmetric superconducting quantum interference devices connected by a π junction. Each qubit in Ref. [10] is controlled via two different frequencies of microwave, while we here only need one of them.

(4) It is not a "step by step" one, thus its scalability is better than those in Refs. [14, 15]. In our scheme, since the time

needed for a single qubit operation is negligibly short in comparison with the time needed for the multipartite collective operation, which is independent of the number of qubits involved, the total time needed for generating the cluster states is almost independent on the number of the qubits.

(5) In our scheme, the couplings among different qubits are mediated by the cavity field, and thus no auxiliary devices are needed, noting that auxiliary devices would increase the complexity of the circuits and inevitably introduce additional uncontrollable noises. Besides, the coupling of any qubit to the cavity could be easily switched on and off via the control of the external gate voltage and the flux of the microwave. In addition, the cavity-qubit coupling is insensitive to the thermal state of the cavity mode by removing the influence of the cavity mode via the periodical evolution.

Before concluding the paper, we briefly address the experimental feasibility of the proposed scheme with the parameters already available in current experimental setups. Suppose that the quality factor of the superconducting cavity is $Q = 1\times10^6$ [20], for the cavity with $\hbar\omega_c = 30\mu ev$ ($\omega_c = 30$GHZ)[8], the cavity decay time is $\tau = Q/\omega_c \approx 33\mu$s. The decoherence time of qubit without the protection of the cavity is $T_d \approx 0.5\mu$s [21].From Eq. (15), we estimate $\delta \approx 0.6$GHZ for $g = 10^{-2}$ [6], $E_J = 40\mu$ev [3, 4], $k = 1$ and $n = 0$. The time for single qubit rotation without cavity is $t_1 \approx 3.3\times(N-1)$ps for $\gamma = \pi/8$ from Eq. (16), and $T \approx 10$ns from Eq. (9). Thus the total manipulation time $t_k = t_1 + T \approx 10$ns, which is much shorter than the cavity decay time τ and the decoherence time of qubit T_d. With the vacuum Rabi frequency $\Omega = 15$MHz and the lifetime of the qubit $\gamma = 2\mu$s, the strong coupling limit can be readily fulfilled ($\Omega^2\tau\gamma \sim 10^4 \gg 1$). So, with the properly chosen parameters, both Lamb-Dicke and strong coupling limits can be fulfilled simultaneously.

In summary, we have proposed a new simple scheme for implementing the multi-qubit operations with superconducting charge qubits coupled to a microwave cavity mode. The quantum operations depend only on global geometric features and are insensitive to the thermal state of the cavity mode, and thus it may result in high-fidelity quantum information processing. In particular, we have illustrated how to generate the highly entangled cluster state more efficiently in the present solid-state system without auxiliary devices, which is promising for realizing one-way quantum computation.

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